Proceedings of the Jangjeon Mathematical Society
20 (2017), No. 3. pp. 483 – 503

www.jangjeon.or.kr
http://dx.doi.org/10.17777/pjms2017.20.3.483

An improved method based on Legendre computational matrix method for time dependent Michaelis-Menten enzymatic reaction model arising in mathematical chemistry

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Abstract

A mathematical model of dynamic form of the Michaelis-Menten enzymatic reaction model is discussed. In this paper, we have applied Legendre spectral algorithm for solving the time dependent Michaelis-Menten enzymatic reaction equations. To the best of our knowledge until there is no rigorous Legendre wavelet solution has been reported for the above mentioned model. From the Legendre spectral solutions, we are then able to analyze the efficiency of the enzymatic reaction model parameters on the solutions to the dynamic Michaelis-Menten enzymatic reaction equations. The numerical results demonstrate the accuracy and efficiency of the proposed spectral approach. The Legendre computational matrix method (LCMM) is shown to be a rather useful and efficient tool for constructing analytical solutions to the dynamic Michaelis-Menten enzymatic reaction equations. Convergence analysis of the proposed method is discussed. Some illustrative examples are given to demonstrate the validity and applicability of the proposed method.

Keyword: Dynamic Michaelis-Menten model; Nonlinear dynamics; Enzyme reaction; Legendre computational matrix; Homotopy analysis method; Runge-Kutta-Felhberg method.

MSC Subject Classification (2010): 42C05, 42C40, 34K28, 78M22, 34B18

1 Introduction

The dynamical form of the Michaelis-Menten model is extensively applied in the theoretical study of enzyme kinetics reactions [17].

$$\frac{dS}{dt} = -k_1 ES + k_{-1}C, \tag{1}$$

$$\frac{dE}{dt} = -k_1 ES + (k_{-1} + k_2)C, \tag{2}$$

$$\frac{dC}{dt} = k_1 ES - (k_{-1} + k_2)C, \tag{3}$$

$$\frac{dP}{dt} = k_2 C, \tag{4}$$

Where S(t) is the concentration of a substance, E(t) is the concentration of an enzyme, C(t) is the concentration of the resulting complex, and P(t) is the concentration of the resulting product. From the framework, a substrate S reacts with an enzyme E to form a complex C which is in turn converted into a product P and the enzyme E; the schematic is

$$E + S \rightleftharpoons C \longrightarrow E + P$$

Note that
(i)$k_1 > 0$ is the rate of reaction governing the production of the complex from the substrate and the enzyme,
(ii)$k_{-1} > 0$ is the rate of reaction governing decomposition of the complex to the substrate and enzyme, and
(iii)$k_2 > 0$ is the rate of reaction governing the breakdown of the complex into the product and the enzyme.
Let us label the initial conditions as

$$S(0) = S_0, \quad E(0) = E_0, \quad C(0) = 0 \quad and \quad P(0) = 0$$

System of Michaelis-Menten equations arise in several physical phenomena, such as pattern formation, chemical reaction etc. Recently, Kristina Mallory and Van Gorder [11] established the variational approach for solving the time-dependent differential equation. Vogt [31] used the homotopy perturbation method (HPM) for the analytical solutions of differential equations in enzyme kinetics. Uma Maheswari and Rajendran [30] had applied the Homotopy Perturbation method (HPM) for the analytical solutions of non-linear reaction-diffusion equations containing a non-linear term related to enzymatic reaction arising in mathematical chemistry. Rasi et al. [23] had showed the approximate analytical expressions for the steady-state concentration of substrate and co-substrate over amperometric biosensors for different enzyme kinetics. Sivasankari et al. [28] had established the Adomian decomposition method (ADM) for the steady state reaction-diffusion equations. Manimozhi et al. [15] have introduced the He's variational iteration method (VIM) and homotopy perturbation method (HPM) for solving the steady-state substrate concentration in the action of biosensor response at mixed enzyme kinetics. Petr Kuzmic et al. [19] introduced an algebraic model for the kinetics of covalent enzyme inhibition at low substrate concentrations. Praveen et al. [20] established the theoretical analysis of intrinsic reaction kinetics and the behavior of immobilized enzymes system for steady-state conditions. Seyyed Ali Madani Tonekaboni et al. [27] had established the analytical solution of substrate concentration in the biosensor response by the Homotopy analysis method (HAM). Kenneth Johnson [10] reviewed the historical development and logical progression of methods for quantitative analysis of enzyme kinetics from 1913 to till date.

Abu-Reesh [1] derived the analytical solutions for the optimal design of a number of membrane reactors in series performing enzyme catalyzed reactions described by Michaelis-Meten kinetics with competitive product inhibition. Golicnik [16] established the numerical solutions to a Michaelis-Menten model in terms of the Lambert W(x) function. Previously, the homotopy perturbation method has been applied to the study of enzyme reaction models [15]. Rahamathunissa and Rajendran [21] had established the He's variational iteration method (VIM) for Michaelis-Menten kinetic of the enzymatic reactions. Also they used the VIM for saturated catalytic kinetics and unsaturated (first order) catalytic kinetics. Thiagarajan et al. [29] had applied the homotopy perturbation method (HPM) to find the analytical solution of the steady-state catalytic current of mediated bio-catalysis. Further, based on the outcome of their work it is possible to compute the approximate amounts of mediator concentration and current corresponding to a nonlinear Michaelis-Menten kinetics scheme. Ronald Li et al. [25] used the optimal homotopy analysis method (HAM) for the time-dependent Michaelis-Menten enzymatic reaction model. Loghambal and Rajendran [12] had introduced the analytical expressions for steady-state concentration of substrate and oxidized and reduced mediator in an amperometric biosensor. Recently, Anandan Anitha et al. [3] have applied the homotopy perturbation method (HPM) to describe the behavior of amperometric biosensor at mixed oxidate enzymatic reaction model.

Wavelets theory is a relatively new and an emerging area in the field of applied science and engineering. It has been applied in a wide range of engineering disciplines; particularly, wavelets are very successfully used in signal analysis for waveform representation and segmentation, time-frequency analysis and fast algorithms for easy implementation [9]. Wavelets permit the accurate representation of a variety of functions and operators. Moreover wavelets establish a connection with fast numerical algorithms. Many researchers started using various wavelets for analyzing problems of high computational complexity. It is proved that wavelets are powerful tool to explore new directional in solving differential equations and integral equations [8].

Due to the nonlinear boundary value problem (NBVPs), analytical solutions of NBVPs are usually difficult to obtain. Consequently,different methods have been developed to give numerical solutions for NBVPs, such as finite difference method, Laplace transform method, Adomian decomposition method, Variational iteration method, differential transform method,operational approach etc. Moreover, orthogonal functions also play an important role in finding numerical solutions for NBVPs, such as block pulse functions, Bernstein polynomials, shifted Legendre polynomials, Chebyshev wavelets, Legendre wavelets etc. Most of the relevant references constructed some operational matrices in order to transform NBVPs into a series of linear or nonlinear algebraic equations. Recently, Hariharan and Kannan [9] have reviewed the wavelet methods for the solution of reaction-diffusion problems in science and engineering. This review shows that the wavelet based method is an efficient and powerful in solving wide class of linear and nonlinear reaction-diffusion equations. Some excellent references therein. Chen and Hsiao [4] introduced the Haar wavelet method [HWM] to lumped and distributed parameter system. Hariharan and his co-workers [7,8] had introduced the wavelet methods to differential equations arising in science and engineering. Recently, Hariharan [6] introduced the Legendre wavelet based hybrid method to film-pore diffusion model. Mahalakshmi and Hariharan [13] used the Chebyshev wavelet method to steady state reaction-diffusion problems arising in mathematical chemistry. Albert Goldbeter [2] reviewed the oscillatory enzyme reactions and Michaelis-Menten kinetics. A specific example is discussed in this review paper and show how the Michaelis-Menten equation, initially introduced for determining the kinetic properties of isolated enzymes, can be applied for studying the dynamic behavior of enzyme system. Inspired by [6],this paper aims at direct method for solving the nonlinear differential equations by using Legendre wavelets functions and operational matrices. Legendre wavelets functions have several advantages like: a)orthogonality; b)simple form; and c)easy to use.

This paper is organized as follows. In section 2, the mathematical formulation of the problem is presented. Method of solution by the Legendre computational matrix method (LCMM) is presented in section 3. In section 4, multi-resolution analysis is presented. Illustrative examples are given in section 5. Results and discussion are given in section 6. Concluding remarks are given in section 7.

2 Background and derivation of the governing equation

Consider the Mallory and Van Gorder [14] which inputs the system into a single second order nonlinear ordinary differential equation. For completeness,we have

$$\frac{dE}{dt} = (k_1 E(t)+k_{-1}+k_2)(E_0-E(t))-k_1 S_0 E(t)+k_1 k_2 E(t)\int_0^t (E_0-E(\tau))d\tau \quad (5)$$

This is an equation with a derivative and an integral in the unknown function E(t). To remove the non-locality inherent in having an integral term, we may define

$$G(t) = \int_0^t (E_0 - E(\tau))d\tau \quad (6)$$

Then

$$G^{'}(t) = E_0 - E(t) \quad and \quad G^{"}(t) = -E^{'}(t) \quad (7)$$

Eq.(10) of Mallory and Van Goder[14] reduces to an equation of the form

$$\frac{1}{k_1}G" + (E_0 + S_0 + K_m)G^{'} + k_1k_2E_0G - k_1G'^2 - k_1k_2GG' = k_1E_0S_0 \quad (8)$$

where

$$K_m = \frac{k_{-1} + k_2}{k_1} \quad (9)$$

is the Michaelis-Menten constant. Making the change of variables

$$G(t) = \frac{g(T)}{k_1}, \quad T = k_1t \quad (10)$$

We have

$$g'' + (E_0 + S_0 + K_m)g' + \frac{k_2E_0}{k_1}g - g'^2 - \frac{k_2}{k_1}gg' = E_0S_0 \quad (11)$$

For the initial conditions, note that

$$G(0) = \int_0^0 (E_0 - E(\tau))d\tau = 0 \quad and \quad G'(0) = E_0 - E(0) = E_0 - E_0 = 0 \quad (12)$$

Then g(0)=0 and g'(0)=0. Hence we shall solve the initial value problem

$$g" + (E_0 + S_0 + K_m)g' + \frac{k_2E_0}{k_1}g - g'^2 - \frac{k_2}{k_1}gg' = E_0S_0 \quad (13)$$

$$g(0) = 0 \quad and \quad g'(0) = 0$$

Regarding the homotopy treatment, we should define the nonlinear operator by

$$N[g] = g" + (E_0 + S_0 + K_m)g' + \frac{k_2E_0}{k_1}g - g'^2 - \frac{k_2}{k_1}gg' - E_0S_0 \quad (14)$$

The auxiliary linear operator should be chosen so that solutions decay as $T \to \infty$
One may recover the meaningful solution by

$$E(t) = E_0 - G'(t) = E_0 - \frac{dg(k_1t)}{dt(k_1)} = E_0 - g'(k_1t) \quad (15)$$

Note that
$E(0) = E_0(g'(0) = 0)$ and $E(t) \to E_0$ as $t \to \infty$ (if $g' \to 0$ as $T \to \infty$).

For reasonable parametric values, the function g' should attain a maximal value and then decay back to zero for large $T\cdot$ This is consistent with the numerical results in the literature [25].
Once g is found, we can recover the other quantities of interest, which can be found by

$$C(t) = g'(k_1t),$$

$$S(t) = S_0 - g'(k_1t) + \frac{k_2}{k_1}g(k_1t),$$

$$P(t) = \frac{k_2}{k_1}g(k_1t)\cdot \quad (16)$$

From these solutions, we have the relation

$$S(t) + C(t) = S_0 + P(t)\cdot \tag{17}$$

3 Wavelets and Legendre wavelets [24]

Wavelets constitute a family of functions constructed from dilation and translation of a single function called the mother wavelet. When the dilation parameter a and the translation parameter b vary continuously we have the following family of continuous wavelets as [9]

$$\psi_{a,b}(t) = |a|^{-\frac{1}{2}}\psi\left(\frac{t-b}{a}\right), \quad a, b \in \mathbb{R}, a \neq 0 \tag{18}$$

If we restrict the parameters a and b to discrete values as $a = a_0^{-k}, b = nb_0a_0^{-k}, a_0 > 1, b_0 > 0$ and n, and k positive integers, we have the following family of discrete wavelets:

$$\psi_{k,n}(t) = |a_0|^{\frac{k}{2}}\psi(a_0^k t - nb_0),$$

where $\psi_{k,n}(t)$ forms a wavelet basis for $\mathcal{L}^2(\mathbb{R})$.
In particular, when $a_0 = 2$ and $b_0 = 1$, then $\psi_{k,n}(t)$ forms an orthonormal basis [9].
Legendre wavelets $\psi_{n,m}(t) = \psi(k, \hat{n}, m, t)$ have four arguments; $\hat{n} = 2n - 1$, $n = 1, 2, 3, \ldots, 2^{k-1}, k$ can assume any positive integer, m is the order for Legendre polynomials and t is the normalized time. They are defined in the interval $[0, 1]$ as

$$\psi_{n,m}(t) = \begin{cases} \sqrt{m+\frac{1}{2}}\,2^{\frac{k}{2}}P_m(2^k t - \hat{n}), & \text{for } \frac{\hat{n}-1}{2^k} \leq t \leq \frac{\hat{n}+1}{2^k} \\ 0, & \text{for } \textit{otherwise} \end{cases} \tag{19}$$

where $m = 0, 1, \ldots, M - 1, n = 1, 2, \ldots, 2^{k-1}$. In Eq.(19), the coefficient $\sqrt{m+\frac{1}{2}}$ is for orthonormality, the dilation parameter is $a = 2^{-k}$ and translation parameter is $b = \hat{n}2^{-k}$. Here, $P_m(t)$ are the well-known Legendre polynomials of order m which are orthogonal with respect to the weight function $w(t) = 1$ on the interval $[-1, 1]$, and satisfy the following recursive formula:

$$P_0(t) = 1, P_1(t) = t,$$

$$P_{m+1}(t) = \left(\frac{2m+1}{m+1}\right)tP_m(t) - \left(\frac{m}{m+1}\right)P_{m-1}(t), \quad m = 1, 2, 3, \ldots$$

3.1 Function approximation [24]

A function $f(t)$ defined over $[0, 1)$ may be expanded as

$$f(t) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{n,m}\psi_{n,m}(t), \tag{20}$$

where $c_{n,m} = (f(t), \psi_{n,m}(t))$ in which $(,)$ denotes the inner product.
If the infinite series in Eq.(20) is truncated, then Eq.(20) can be written as

$$f(t) \simeq \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(t) = C^T\Psi(t), \tag{21}$$

where C and $\Psi(t)$ are $2^{k-1}M \times 1$ matrices given by

$$C = [c_{10}, c_{11}, \dots, c_{1M-1}, c_{20}, \dots, c_{2M-1}, \dots, c_{2^{k-1}0}, \dots, c_{2^{k-1}M-1}]^T$$

$$\Psi(t) = [\psi_{10}(t), \psi_{11}(t), \dots, \psi_{1M-1}(t), \psi_{20}(t), \dots, \psi_{2M-1}(t), \dots, \psi_{2^{k-1}0}(t), \dots, \psi_{2^{k-1}M-1}(t)]^T \tag{22}$$

The integration of the vector $\Psi(t)$ defined in Eq.(22) can be obtained as

$$\int_0^t \Psi(t')dt' = P\Psi(t), \tag{23}$$

where P is the $(2^{k-1}M) \times (2^{k-1}M)$ operational matrix for integration and is given in [24] as

$$P = \frac{1}{2^k}\begin{bmatrix} L & F & F & \dots & F \\ O & L & F & \dots & F \\ \vdots & O & \ddots & \ddots & \vdots \\ & & & & F \\ O & O & \dots & O & L \end{bmatrix} \tag{24}$$

In Eq.(24) F and L are $M \times M$ matrices given by

$$F = \begin{bmatrix} 2 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix},$$

and

$$L = \begin{bmatrix} 1 & \frac{1}{\sqrt{3}} & 0 & 0 & \dots & 0 & 0 & 0 \\ -\frac{\sqrt{3}}{3} & 0 & \frac{\sqrt{3}}{3\sqrt{5}} & 0 & \dots & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{5}}{5\sqrt{3}} & 0 & \frac{\sqrt{5}}{5\sqrt{7}} & \ddots & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{7}}{7\sqrt{5}} & 0 & \ddots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & -\frac{\sqrt{2M-3}}{(2M-3)\sqrt{2M-5}} & 0 & \frac{\sqrt{2M-3}}{(2M-3)\sqrt{2M-1}} \\ 0 & 0 & 0 & 0 & \dots & 0 & -\frac{\sqrt{2M-1}}{(2M-1)\sqrt{2M-3}} & 0 \end{bmatrix}$$

The integration of the product of two Legendre wavelet function vectors is obtained as

$$I = \int_0^1 \Psi(t)\Psi^T(t)dt \tag{25}$$

where I is an identity matrix.

3.2 Operational matrix of derivative and product operation matrix

In the following section, we introduce a new Legendre wavelets operational matrix of derivative.
Theorem 2. Let $\psi(t)$ be the Legendre wavelets vector defined in Eq.(22)
The derivative of the vector $\psi(t)$ can be expressed by

$$\frac{d\psi(t)}{dt} = D\psi(t) \tag{26}$$

where D is the $2^k(M+1)$ operational matrix of derivative defined as follows

$$D = \begin{bmatrix} F & 0 & \dots & 0 \\ 0 & F & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & F \end{bmatrix},$$

in which F is $(M+1)\times(M+1)$ matrix and its (r,s)th element is defined as follows.

$$F_{r,s} = \begin{cases} 2^{k+1}\sqrt{(2r-1)(2s-1)} \quad r = 2,...,(M+1), s = 1,...,r-1 & \text{for} \quad (r+s)odd, \\ 0, & \text{for} \quad otherwise \end{cases} \tag{27}$$

Proof: See Ref.[18]
Corollary. By using Eq.(26) the operational matrix for nth derivative can be derived as

$$\frac{d^n\psi(t)}{dx^n} = D^n\psi(t) \tag{28}$$

Where D^n is the nth power of matrix D.

4 Applications of the operational matrix of derivative

4.1 Linear differential equation

Consider the linear second order differential equation

$$y"(x) + f_1(x)y'(x) + f_2(x)y(x) = g(x) \tag{29}$$

with the initial conditions

$$y(0) = A, \quad y'(0) = B, \tag{30}$$

or boundary conditions

$$y(0) = A, \quad y'(1) = B. \tag{31}$$

To solve problem (29) we approximate $y(x), f_1(x), f_2(x)$ and $g(x)$ by the Legendre wavelets as

$$\begin{aligned} y(x) &= C^T\psi(x), \\ f_1(x) &= F_1^T\psi(x), \\ f_2(x) &= F_2^T\psi(x), \\ g(x) &= G^T\psi(x) \end{aligned} \tag{32}$$

By using Eq.(28) and (32) we have

$$y'(x) = C^T D\psi(x), \quad y"(x) = C^T D^2\psi(x) \tag{33}$$

Employing Eq.(32) and (33), the residual $R(x)$ for Eq.(29) can be written as

$$R(x) = (C^T D^2\psi(x) + F_1^T\psi(x)\psi^T(x)D^T C + F_2^T\psi(x)\psi^T(x)C - G^T\psi(x)) \tag{34}$$

By using the product operation matrix of Legendre wavelets, we have

$$R(x) = (\psi^T(x)(D^2)^T C + \psi^T(x)\tilde{F}_1 D^T C + \psi^T(x)\tilde{F}_2 C - \psi^T(x)G). \tag{35}$$

As in a typical Tau method, we generate $2^k(M+1) - 2$ linear equations by applying

$$\int_0^1 \Psi_j(x)R(x)dx = 0 \quad j = 1, ...2^k(M+1) - 2 \tag{36}$$

Also, by substituting the initial conditions (30) in Eq.(32)-(33) we have

$$\begin{aligned} y(0) &= C^T\psi(0) = A, \\ y'(0) &= C^T D\psi(0) = B. \end{aligned} \tag{37}$$

Or for boundary value problems we have

$$y(0) = C^T\psi(0) = A, y(1) = C^T\psi(1) = B \tag{38}$$

Eq.(36)-(38) generate $2^k(M+1)$ set of linear equations. These linear equations can be solved for unknown coefficients of the vector C. Consequently, y(x) given in Eq.(32) can be calculated.

5 Multi-resolution analysis (MRA)

The best way to understand the wavelets is through a multi-resolution analysis. Given a function $f \in \mathcal{L}^2(\mathbb{R})$ a multi-resolution analysis (MRA) of $\mathcal{L}^2(\mathbb{R})$ produces a sequence of subspaces $V_j, V_{j+1}, \ldots$ such that the projections of f onto these spaces give finer and finer approximations of the function f as $j \to \infty$.

5.1 Definition 1.(Multi-resolution analysis)

A multi-resolution analysis of $\mathcal{L}^2(\mathbb{R})$ is defined as a sequence of closed subspaces $V_j \subset \mathcal{L}^2(\mathbb{R}), j \in \mathbb{Z}$ with the following properties

- $\cdots \subset V_{-1} \subset V_0 \subset V_1 \subset \ldots$
- The space V_j satisfy $\bigcup_{j\in\mathbb{Z}} V_j$ is dense in $\mathcal{L}^2(\mathbb{R})$ and $\bigcap_{j\in\mathbb{Z}} V_j = 0$
- If $f(x) \in V_0, f(2^j x) \in V_j$, i.e. the spaces V_j are scaled versions of the central space V_0
- If $f(x) \in V_0, f(2^j x - k) \in V_j$, i.e. all the V_j are invariant under translation
- There exists $\Phi \in V_0$ such that $\Phi(x-k); k \in \mathbb{Z}$ is a Riesz basis in V_0

6 Convergence analysis

6.1 Theorem 1

The series solution $y(x) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{n,m}\psi_{n,m}(x)$, defined in Eq.(20) using Legendre wavelet method converges to $y(x)$.

Proof. See Ref.[26]

7 Illustrative examples

7.1 Example

Consider the nonlinear initial value problem

$$g'' + (E_0 + S_0 + K_m)g' + \frac{k_2 E_0}{k_1} g - g'^2 - \frac{k_2}{k_1} g g' = E_0 S_0 \tag{39}$$

with the following initial conditions $g(0) = 0$ and $g'(0) = 0$

The suggested method is applied with M=2, and the solution g(t) is approximated as follows

$$K_m = \frac{k_{-1} + k_2}{k_1} \tag{40}$$

We solve the Eq.(40) using the algorithm described in Properties of Legendre Wavelets section for the case corresponds to M=2, k=0 to obtain an approximate solution of u(x). First, if we make use of Eq.(41), then the two operational matrices D and D^2 are given by

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 6 & 0 \end{bmatrix},$$

$$D^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 12 & 0 & 0 \end{bmatrix},$$

$$\Psi(t) = \sqrt{\frac{2}{\pi}} \begin{bmatrix} 1 \\ 2t-1 \\ 6t^2-6t+1 \end{bmatrix},$$

$$C^T = \sqrt{\frac{\pi}{2}} \begin{bmatrix} c_0 & c_1 & c_2 \end{bmatrix}$$

For M=2, a system of three nonlinear algebraic equations is obtained, two of them from the initial conditions and the other from the main equation using the collocation point x=0.5;

Case(i) Consider $k_{-1} = k_1 = k_2 = 1$ and $E_0 = S_0 = 1$

$$g" + g'(4-g) - g'^2 + g - 1 = 0 \tag{41}$$

$$C^T D^2 \psi(t) + C^T D\psi(t)[4 - C^T\psi(t)] - [C^T D\psi(t)]^2 + C^T\psi(t) - 1 = 0$$

which is equivalent to

$$12c_2 + 8c_1 - 2c_0c_1 + c_1c_2 - 4{c_1}^2 + c_0 - 0.5c_2 - 1 = 0 \tag{42}$$

Furthermore, the use of initial conditions in Eq.(40) lead to the two equation,

$$g(0) = 0 \Rightarrow c_0 = 2c_2 \tag{43}$$

$$g'(0) = 0 \Rightarrow c_1 = 3c_2 \tag{44}$$

The solution of the nonlinear system of equations Eq.(41)-(44) give

$$c_0 = 0.0552, \quad c_1 = 0.0828, \quad c_2 = 0.0276$$

Consequently,

$$g(t) = \begin{pmatrix} 0.0552 & 0.0828 & 0.0276 \end{pmatrix} \begin{pmatrix} 1 \\ 2t-1 \\ 6t^2-6t+1 \end{pmatrix}$$

$$g(t) = 0.1656t^2 \tag{45}$$

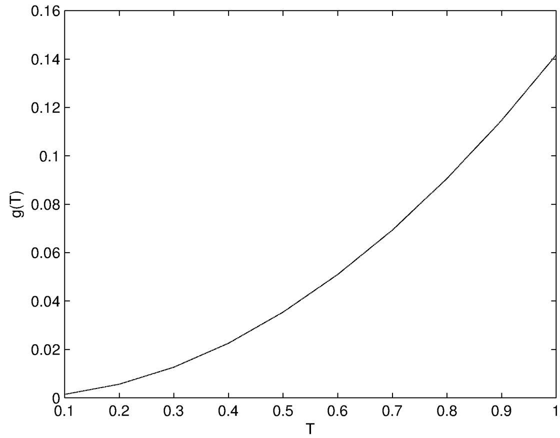

Figure 1: Computation of g(t) for various t values

Case(ii) Consider $k_{-1} = 0.5, k_1 = 1, k_2 = 0.5$ and $E_0 = S_0 = 1$

$$g" + g'(3 - 0.5g) - g'^2 + 0.5g - 1 = 0 \tag{46}$$

$$C^T D^2 \psi(t) + C^T D\psi(t)[3 - 0.5C^T \psi(t)] - [C^T D\psi(t)]^2 + 0.5C^T \psi(t) - 1 = 0$$

which is equivalent to

$$12c_2 + 6c_1 - c_0c_1 + 0.5c_1c_2 - 4{c_1}^2 + 0.5c_0 - 0.25c_2 - 1 = 0 \tag{47}$$

Furthermore, the use of initial conditions in Eq.(47) lead to the two equation,

$$g(0) = 0 \Rightarrow c_0 = 2c_2 \tag{48}$$
$$g'(0) = 0 \Rightarrow c_1 = 3c_2 \tag{49}$$

The solution of the nonlinear system of equations Eq.(47)-(49) give

$$c_0 = 0.068, \quad c_1 = 0.102, \quad c_2 = 0.0340$$

Consequently,

$$g(t) = \begin{pmatrix} 0.068 & 0.102 & 0.034 \end{pmatrix} \begin{pmatrix} 1 \\ 2t - 1 \\ 6t^2 - 6t + 1 \end{pmatrix}$$

$$g(t) = 0.204t^2 \tag{50}$$

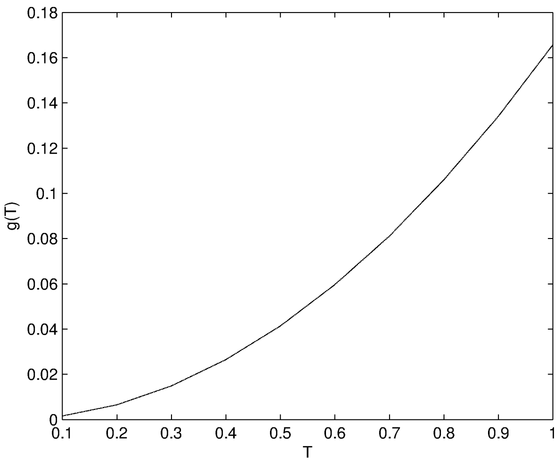

Figure 2: Computation of g(t) for various t values

Case(iii) Consider $k_{-1} = 1, k_1 = 1, k_2 = 0.5$ and $E_0 = S_0 = 1$

$$g" + g'(3.5 - 0.5g) + g'^2 + 0.5g - 1 = 0 \tag{51}$$

$$C^T D^2\psi(t) + C^T D\psi(t)[3.5 - 0.5C^T\psi(t)] + [C^T D\psi(t)]^2 + 0.5C^T\psi(t) - 1 = 0$$

which is equivalent to

$$12c_2 + 7c_1 + 0.5c_0 - 0.25c_2 + 4{c_1}^2 - c_0c_1 + 0.5c_2c_1 - 1 = 0 \tag{52}$$

Furthermore, the use of initial conditions in Eq.(51) lead to the two equation,

$$g(0) = 0 \Rightarrow c_0 = 2c_2 \tag{53}$$

$$g'(0) = 0 \Rightarrow c_1 = 3c_2 \tag{54}$$

The solution of the nonlinear system of equations Eq.(52)-(54) gives

$$c_0 = 0.0578, \quad c_1 = 0.0867, \quad c_2 = 0.0289$$

Consequently,

$$g(t) = \begin{pmatrix} 0.0578 & 0.0867 & 0.0289 \end{pmatrix} \begin{pmatrix} 1 \\ 2t - 1 \\ 6t^2 - 6t + 1 \end{pmatrix}$$

$$g(t) = 0.1734t^2 \tag{55}$$

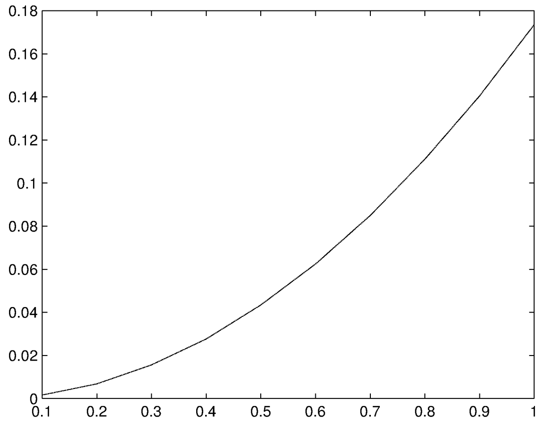

Figure 3: Computation of g(t) for various t values

Case(iv) Consider $k_{-1} = 0.5, k_1 = 1 = k_2 = 1$ and $E_0 = S_0 = 1$

$$g" + g'(3.5 - g) - g'^2 + g - 1 = 0 \tag{56}$$

$$C^T D^2\psi(t) + C^T D\psi(t)[3.5 - C^T\psi(t)] - [C^T D\psi(t)]^2 + C^T\psi(t) - 1 = 0$$

which is equivalent to

$$12c_2 + 7c_1 + c_0 - 0.5c_2 + 4{c_1}^2 - 2c_0c_1 - c_2c_1 - 1 = 0 \tag{57}$$

Furthermore, the use of initial conditions in Eq.(56) lead to the two equation,

$$g(0) = 0 \Rightarrow c_0 = 2c_2 \tag{58}$$

$$g'(0) = 0 \Rightarrow c_1 = 3c_2 \tag{59}$$

The solution of the nonlinear system of equations Eq.(57)-(59) gives

$$c_0 = 0.057, \quad c_1 = 0.0855, \quad c_2 = 0.0285$$

Consequently,

$$g(t) = \begin{pmatrix} 0.057 & 0.0855 & 0.0285 \end{pmatrix} \begin{pmatrix} 1 \\ 2t-1 \\ 6t^2 - 6t + 1 \end{pmatrix}$$

$$g(t) = 0.171t^2 \tag{60}$$

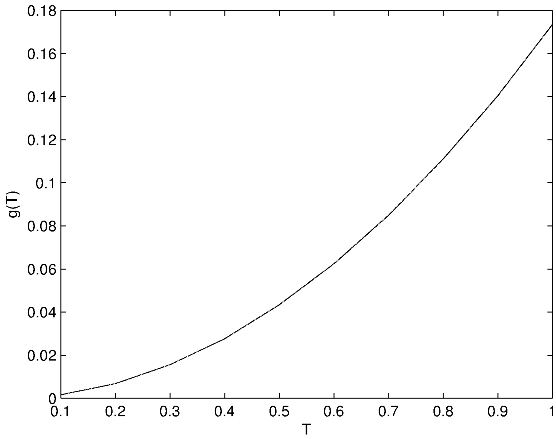

Figure 4: Computation of g(t) for various t values

Case(v) Consider $k_{-1} = k_1 = 0.5, k_2 = 1$ and $E_0 = S_0 = 1$

$$g" + g'(5 - 2g) - g'^2 + 2g - 1 = 0 \tag{61}$$

$$C^T D^2\psi(t) + C^T D\psi(t)[5 - 2C^T\psi(t)] - [C^T D\psi(t)]^2 + 2C^T\psi(t) - 1 = 0$$

which is equivalent to

$$12c_2 + 10c_1 + 2c_0 - c_2 - 4{c_1}^2 - 4c_0c_1 + 2c_2c_1 - 1 = 0 \tag{62}$$

Furthermore, the use of initial conditions in Eq.(61) lead to the two equation,

$$g(0) = 0 \Rightarrow c_0 = 2c_2 \tag{63}$$
$$g'(0) = 0 \Rightarrow c_1 = 3c_2 \tag{64}$$

The solution of the nonlinear system of equations Eq.(62)-(64) gives

$$c_0 = 0.0456, \quad c_1 = 0.0684, \quad c_2 = 0.0228$$

Consequently,

$$g(t) = \begin{pmatrix} 0.0456 & 0.0684 & 0.0228 \end{pmatrix} \begin{pmatrix} 1 \\ 2t - 1 \\ 6t^2 - 6t + 1 \end{pmatrix}$$

$$g(t) = 0.1368t^2 \tag{65}$$

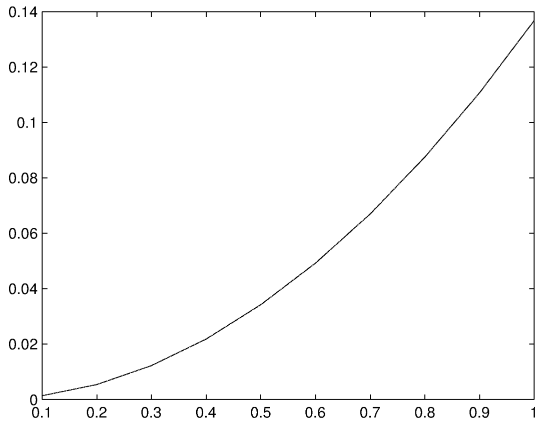

Figure 5: Computation of g(t) for various t values

Case(vi) Consider$k_{-1} = 1, k_1 = 0.5, k_2 = 1$ and $E_0 = S_0 = 1$

$$g" + g'(6 - 2g) - g'^2 + 2g - 1 = 0 \tag{66}$$

$$C^T D^2\psi(t) + C^T D\psi(t)[6 - 2C^T\psi(t)] - [C^T D\psi(t)]^2 + 2C^T\psi(t) - 1 = 0$$

which is equivalent to

$$12c_2 + 12c_1 + 2c_0 - c_2 - 4{c_1}^2 - 4c_0c_1 + 2c_2c_1 - 1 = 0 \tag{67}$$

Furthermore, the use of initial conditions in Eq.(66) lead to the two equation,

$$g(0) = 0 \Rightarrow c_0 = 2c_2 \tag{68}$$

$$g'(0) = 0 \Rightarrow c_1 = 3c_2 \tag{69}$$

The solution of the nonlinear system of equations Eq.(67)-(69) gives

$$c_0 = 0.0400, \quad c_1 = 0.0600, \quad c_2 = 0.0200$$

Consequently,

$$g(t) = \begin{pmatrix} 0.0400 & 0.0600 & 0.0200 \end{pmatrix} \begin{pmatrix} 1 \\ 2t - 1 \\ 6t^2 - 6t + 1 \end{pmatrix}$$

$$g(t) = 0.12t^2 \tag{70}$$

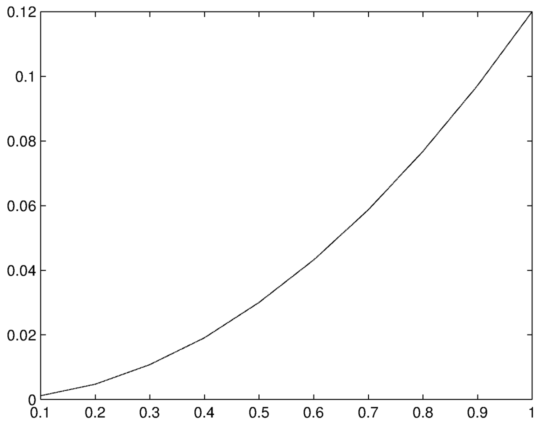

Figure 6: Computation of g(t) for various t values

Case(vii) Consider $k_{-1} = 1, k_1 = k_2 = 0.5$ and $E_0 = S_0 = 1$

$$g" + g'(5 - g) - g'^2 + g - 1 = 0 \tag{71}$$

$$C^T D^2\psi(t) + C^T D\psi(t)[5 - C^T\psi(t)] - [C^T D\psi(t)]^2 + C^T\psi(t) - 1 = 0$$

which is equivalent to

$$12c_2 + 10c_1 + c_0 - 0.5c_2 - 4{c_1}^2 - 2c_0c_1 + c_2c_1 - 1 = 0 \tag{72}$$

Furthermore, the use of initial conditions in Eq.(71) lead to the two equation,

$$g(0) = 0 \Rightarrow c_0 = 2c_2 \tag{73}$$

$$g'(0) = 0 \Rightarrow c_1 = 3c_2 \tag{74}$$

The solution of the nonlinear system of equations Eq.(72)-(74) gives

$$c_0 = 0.0472, \quad c_1 = 0.0708, \quad c_2 = 0.0236$$

Consequently,

$$g(t) = \begin{pmatrix} 0.0472 & 0.0708 & 0.0236 \end{pmatrix} \begin{pmatrix} 1 \\ 2t - 1 \\ 6t^2 - 6t + 1 \end{pmatrix}$$

$$g(t) = 0.1416t^2 \tag{75}$$

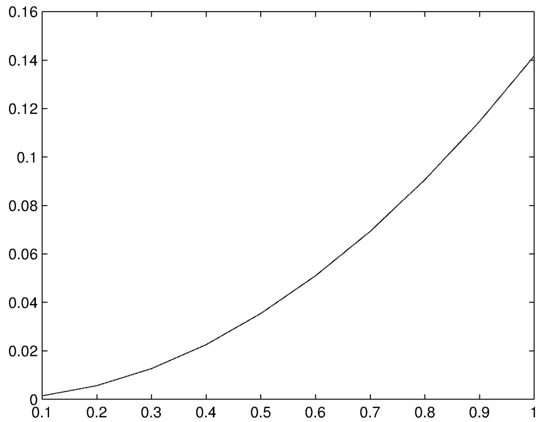

Figure 7: Computation of g(t) for various t values

In figures [1-7] show the legendre wavelet solution for various values of E_0 and S_0. Our results can be compared with Runge-Kutta-Felhberg 4-5 method (denoted RKF45, see[5]) and homotopy analysis method(HAM).In Legendre wavelet method, increasing the value of M we get the results closure to the real values.

8 Conclusion

The dynamical form of the time-dependent Michaelis-Menten enzymatic reaction equation is discussed. From the operational matrix based results shown here, we investigate that the resulting solutions have rather low residual errors after few iterations are calculated, highlighting the accuracy and efficiency of the proposed method. Unlike other methods applied to solve the dynamic Michaelis-Menten kinetic model, the spectral method allows one to control the error inherent in the approximating solutions to the above model.

Acknowledgments
This work was supported by the Department of Science and Technology (DST-SERB, Project No.: SB/FTP/MS-012/2013), Government of India. Our hearty thanks are due to Prof. R. Sethuraman, Vice-Chancellor, SASTRA University, Dr. S. Vaidhyasubramaniam, Dean/Planning and development and Dr. S. Swaminathan, Dean/Sponsored research for their kind encouragement and for providing good research environment.

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